Internship Master 2 level in Geometric Statistics

Université Côte d'Azur, Laboratoire J.A. Dieudonné, CNRS UMR 7351, Nice.

Duration. Between 4 and 6 months, during the second semester of the academic year 2023-2024. Monthly gross salary : around 600€.

Advisors. Elena Di Bernardino, Michele Ancona (Laboratoire J.A. Dieudonné, Nice) and Céline Duval (Laboratoire Paul Painlevé, Lille).

Require qualification. Master 1 or equivalent level in Mathematical Statistics or Geometry.

Required skills.

- Solid background in probability and statistic theory and/or geometry,
- Possible knowledge in spatial statistics,
- Good knowledge in programming languages (as Python, R, C++ or Matlab).

Context. The excursion set at level u of a C^2 smooth d-dimensional random field X, *i.e.*,

$$E_X^T(u) := \{t \in T \,:\, X(t) \geq u\} = T \cap E_X(u), \quad \text{where } E_X(u) := X^{-1}([u, +\infty)),$$

carries relevant information on the field that can be captured through various geometric measures. We consider additive geometric measures that are the Lipschitz-Killing (LK in the sequel) curvatures, denoted $\sigma_\ell(X, u, T),\ \ell = 0, 1, \ldots, d$ (also referred to as intrinsic volumes or Minkowski functionals in the literature). The study of random fields through the geometry of their excursion sets has received a lot of interest. This is mainly stimulated by the applications in medical imaging domain (see for instance Adler and Taylor (2011), Section 5, and the references therein, Di Bernardino and Duval (2022)).

Goal of the internship. During this M2 Internship we will focus on the study of the robustness of the LK estimators to a perturbation of the field X. Recently, Di Bernardino et al. (2022) focus on an isotropic Gaussian random field subject to perturbations of the variance and location of X. Di Bernardino et al. (2020) investigate LK curvatures for excursion sets of random fields on $\mathbb{R}^2$ under a particular perturbation of X which consists in adding a zero mean spatial-invariant random perturbation. *This type of perturbation can be seen as a small super-imposed noise to an underlining Gaussian field.* The authors propose an estimator for the perturbation variance and prove it to be asymptotically normal and unbiased, allowing to make inference by using only the sparse information on the field given by their excursion sets. In this M2 Internship we aim at extending this analysis in the case of spatially varying random perturbations.

This internship can lead to a Ph.D thesis (between October 2024 and October 2027) co-supervised by Elena Di Bernardino, Michele Ancona (Laboratory J.A. Dieudonné, Nice) and Céline Duval (Laboratory Paul Painlevé, Lille).

Needed documents for candidature:

- Detailed Curriculum vitae;
- Motivation letter;
- Notes transcripts of previous academic years (or equivalent);
- At least one name of academic professor for recommendation;

Contacts for candidature. Contacts to apply via mail:

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References

Adler, R. J. and Taylor, J. E. (2011). *Topological complexity of smooth random functions*, volume 2019 of *Lecture Notes in Mathematics*. Springer, Heidelberg. Lectures from the 39th Probability Summer School held in Saint-Flour, 2009,.

Di Bernardino, E. and Duval, C. (2022). Statistics for Gaussian random fields with unknown location and scale using Lipschitz-Killing curvatures. *Scandinavian Journal of Statistics*, 49(1):143–184.

Di Bernardino, E., Estrade, A., and Rossi, M. (2020). On the excursion area of perturbed Gaussian fields. *ESAIM: PS*, 24:252–274.

Di Bernardino, E., Estrade, E., and Opitz, T. (2022). Spatial extremes and stochastic geometry for Gaussian-based peaks-over-threshold processes. *Preprint hal-03825701*.