Degradation model selection for pronostic: application to Wind turbine

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For lifetime prediction or maintenance planning of complex deteriorating systems, degradation modeling in the presence of health monitoring data is essential. Even though, the majority of degradation phenomenons have physical or mechanical meanings (corrosion, erosion, etc.) due to the large number of unknown environmental factors influencing this latter, it is nearly impossible to base the prediction on pure deterministic models. Therefore, the degradation phenomenon can be considered as random with a gradual time-continuous trajectory. With respect to the system under study, the degradation model can take values in discrete or continuous space. For instance, the corrosion indicator can take infinite possible values as soon as it begins but the cumulative number of rejected products in a production day which can be considered as a deterioration indicator is finite and can be enumerated, for more examples refer to [31, 45, 47].

It could be tempting to consider complex models able to take into account all available information and describe precisely the dynamics of degradation. However, these kind of models are not always tractable with large difficulties for inference and calibration in the presence of data. Very simple and tractable models which can be easily calibrated can lead to wrong evaluation of the uncertainty around the lifetime prediction. This wrong evaluation can cause additional costs and desastrous consequences. A fair and satisfactory degradation model should make a balance between accuracy and tractability, [25, 44].

This thesis is devoted to degradation modeling and prognosis in presence of health monitoring data. In presence of data, the main issue is to select the best model which fit data and can describe the underlying degradation phenomenon. Since data is collected for a given system and conditions, it should be manipulated cautiously because it may represent a very specific or extreme behavior of the underlying degradation phenomenon. The best candidate model is the one which takes into account the possibility of extreme behaviors during data collection without losing in perspective the real average degradation behavior. The usage of the degradation model can highly impact how data is tackled and which a model is favoured. If there are safety issues or very high costs concerns, the modeling precision is not considered in the same way. The thesis focuses on pure statistical concerns where the best candidate is derived by efficient statistical tools. For more details and examples, refer to [20, 30]. An efficient statistical tool is able to discard irrelevant models, and if some prior knowledge on the degradation phenomenon is available, the best model between a class of candidate can be proposed. In other words, the aim of degradation modeling is to select a model from a set of competing models capturing the features of the underlying degradation process. As it is mentioned before, the methodology depends mostly on model usage, see for instance [6, 47].

Lévy processes [3] such as Wiener and gamma processes and diffusion processes such as Black and Scholes, Ornstein Uhlembeck processes are commonly an be used to model the degradation. Inference and model calibration for these processes have been widely addressed in the fields of finance, biology and engineering [35, 36]. However, in reliability engineering domain the datasets are smaller and safety constraints are significant concerns [43, 28, 47]. The model selection for reliability engineering prediction problems is an important issue but has not been extensively addressed, [12].

Very often, the health monitoring indicator data are heterogeneous [19, 23]. One of the reasons is due to the heterogeneity of the collection procedure, data could be collected by sensors, periodic or non-periodic inspections. Therefore, the available data is not necessarily identically distributed. For degradation model calibration, goodness of fit tests are required. The efficiency of common goodness of fit test such as Kolmogorov-Smirnov, Anderson-Darling, Cramer von Mises, requires the independent and identically distributed condition, refer to [17, 29, 24]. The existing tests dealing with independent but not identically distributed random variables have, only asymptotical efficiency, which requires large size samples [37, 42, 4, 15, 1, 46]. The independence hypothesis is also a big constraint since many degradation model candidates don't satisfy the condition of independence of increments but only the conditional independance property. Tests applied to conditionnally independent random variables (markovian property) are not always efficient, refer [7, 11, 34, 10, 8]. Moreover, for these processes if the increments are not identically distributed it is very difficult to propose an efficient goodness of fit test. Another difficulty is the temporal aspect of data and non stationarity of the underlying model. Most of the goodness of fit tests for stochastic processes deal with LÃ©vy or stationary processes, see e.g. [11, 2], but for non-stationary or diffusion processes, there are tests for each specific models but it is difficult to find a generic tool for checking the goodness of fit in a set of candidates [13, 9, 16, 38, 22, 5, 21, 40].

The majority of goodness of fit tests use the empirical cumulative distribution function or some order statistics. The problem of non i.i.d. random variables is bypassed by proposing a transformation which leads to a uniformly distributed data. But the dependance does not permit to apply classical tests on the transformed data. A new lead is to propose a transformation which permits to deal with non i.i.d. random variables in a more tractable space [39]. To test the goodness of fit of data related to non i.i.d. random variables and to chose between a class of possible models, it is also possible to consider other metrics proposed in the literature such as divergence, Kullback-Leibler distance, refer to [33, 32]. Another possibility is to use a test based on the famous notions of depth initially introduced by Tukey [41] and largely studied in after on [48, 18, 14, 26, 27]. This indicador, for a multi-dimensional distribution measures the distance between the observations and the median of a distribution. In this case, the median is the deepest point. The depth of a multi-dimensional distribution is well defined and takes into account the geometric form of the distribution domain. A test based on this indicator seems very flexible and can be applied on different kind of data without requiring restrictive hypotheses in comparison to classical goodness of fit tests. These kind of tests are not commonly used for stochastic processes and even less for degradation data.

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